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Separating Efficiency of a Water Elutriator

This paper investigates the separating efficiency of a vertical water elutriator operating on particle mixtures. In particular, comparisons are made with a previously developed mathematical model based on an eddy diffusion mechanism of mixing. Standard mixtures of glass spheres, of uniform size and shape, are used to study the equipment characteristics. It is found that the separating efficiency follows the same trend as predicted by the model and that "wall flow" of particles occurs. This affects the required water velocity, resulting in the equipment producing a top product generally purer than the bottom product.

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SCOPE

The field of waste recycling and the recovery of useful materials from waste will continue to become more important as the supplies of natural resources decrease. Unfortunately, unless some form of source segregation has occurred, the feedstock which presents itself for separation is usually in the form of a complicated mixture of solid particles emerging from a grinder. One of the key separating stages is often water elutriation or air classification, methods which rely on differences in terminal falling velocities between different particles. Although these have often been set up in various forms, little work has been directed towards obtaining a better understanding of their operation, and hence improving their efficiency. A new aspect of the problem in the application to waste streams is that the size and density of the solid particles is such that higher fluid velocities are required than in application to very fine particle mixtures. This introduces fluid turbulence which affects the behavior of the equipment.

The study described in this paper is part of a continuing effort to improve the separating efficiency of this type of equipment. Early work on a small water elutriator is now extended to a much larger-scale model. Standard mixtures of uniform particles are separated. Glass spheres were chosen as requiring similar water velocities to those often encountered in waste recycling operations. A previously developed mathematical model, using an eddy diffusion mechanism superimposed on the expected convective movement of particles, is compared with the actual separation obtained from the equipment.

CONCLUSIONS AND SIGNIFICANCE

The study has demonstrated the presence and significance of the "wall flow" of particles in a vertical water elutriator. This refers to the tendency of particles to cluster together and migrate to the slower moving fluid near the tube wall, rather than remaining near the axis of the tube. This results in the required average fluid velocity being about 5–15% greater than would be expected; the "light" species, trying to move upwards to the top product, can penetrate very significantly downwards to appear in the bottom product. The fluid turbulence means that the "heavy" species can also penetrate upwards towards the top product, and it has been found that this can be predicted satisfactorily using values of eddy diffusivity obtained by Levenspiel for single-phase dispersion coefficients in pipe flow. However, the wall-flow effect means that values of diffusivity of about 2.5 times the Levenspiel value are required for the "light" species penetrating downwards. The optimum separation can be predicted well using this model.

We may conclude that the simple vertical water elutriator is better at producing a pure top product than at producing a pure bottom product; therefore, the point of solids injection should be nearer the top than the bottom. The results give some encouragement that the model will help to clarify the characteristics of this type of equipment, and it is hoped to extend the study to mixtures containing ranges of shapes, densities and sizes, and to relate separating efficiency to the method of grinding the feedstock. Methods of avoiding wall flow are to be studied as well as the application to air classifiers.

INTRODUCTION

A field in which there is great scope for chemical engineers to apply their knowledge is in the recovery and recycling of useful materials from waste. In particular, separation processes are important in the success of most recycling operations, since the feed material usually appears as a very complicated mixture. In the particular area of solids separations, water elutriators and air classifiers are extensively used, applications including paper separation (Colon, 1976), wire stripping waste copper recovery (Jensen et al., 1974), and automobile shredder residues separation (Bilbrey et al., 1978).

Elutriation uses differences in terminal settling velocities among particles to effect a separation. In its simplest form, a water elutriator consists of a vertical tube containing upward flowing water. The mixture of solid particles to be separated is fed into the water stream and some particles move downwards and others move upwards, according to their terminal velocities. However, the application of the technique to the sort of mixtures encountered in waste recycling operations has introduced other factors, which were not so important in earlier applications to mixtures of very small particle sizes. The size and density of many particles being recycled are such that they have quite high terminal velocities, and hence the velocity of the water must be quite high. This introduces turbulence and turbulent mixing into the process, and little work has been directed towards understanding the implications of this. In fact, it seems that many of the elutriators and classifiers described for use in this field have been designed in a very speculative way.

The object of the continuing study described in this paper is to attempt to understand better the mode of operation, and, very importantly, to try to improve separation efficiencies and to relate these efficiencies to the grinding process used to produce the solids mixture. Two previous papers (Biddulph, 1979, 1980) have described early studies on a very small-scale water elutriator. In particular, the way that a "heavy" species, trying to move downwards against the water flow, is mixed upwards by turbulent mixing was the subject of study. This can be so pronounced that some of the "heavy" species can appear in the top product. Similarly, some of the "light" species can be mixed downwards to such an extent that it appears in the bottom product.

This paper describes results obtained on a much larger-scale elutriator, operating on standard mixtures with predictable characteristics, to clarify its characteristics.

The mathematical model which has been established (Biddulph, 1979, 1980) to simulate this behavior uses an eddy mixing mechanism superimposed on the expected net movement of particles.

THEORY

Figure 1 illustrates a slice through the elutriator, a "light" species X is trying to move upwards but is being mixed downwards also. The sign convention is positive downwards. Thus, $Z = 0$ at the top of the elutriator, $Z = Z_F$ at the feed point, and $Z = Z_B$ at the bottom of the elutriator. The liquid flows upwards at V_W m/s, this being negative in this convention.

The net velocity of a particle is given by

$$U_x = U_t + V_W$$

U_x will be negative for a "light" species particle and positive for a "heavy" species particle.

A steady-state mass balance for a species X over the slice illustrated leads to:

$$De\frac{d^2X}{dZ^2} - U_x\frac{dX}{dZ} = 0 \tag{1}$$

The general solution of this equation is

$$X = K_1 e^{U_x Z/De} + K_2 \tag{2}$$

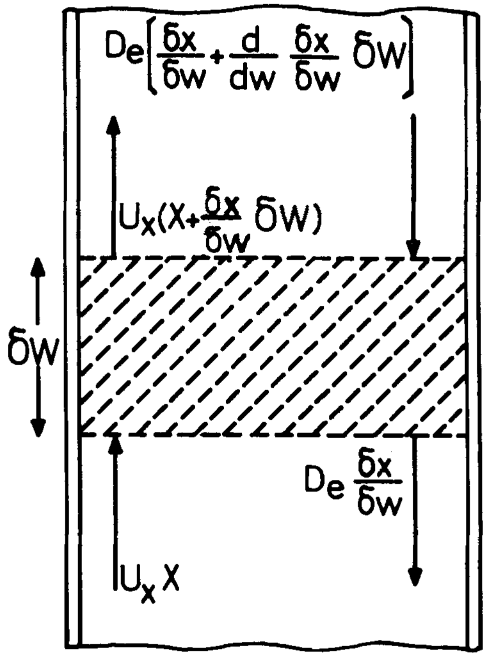

Figure 1. Flows of species X for elutriator model.

This equation applies equally to a 'light" species or a "heavy" species.

For a heavy species, the region of interest from the point of view of establishing whether the model can provide a reasonable representation of reality is that between the feed zone and the top. In this region, any heavy particles appearing would be due to turbulent mixing, since otherwise they would just descend immediately from the feed point.

At the top of the elutriator, $Z = 0$, and Eq. 2 becomes:

$$X_{\mathrm{TOP}} = K_1 + K_2 \tag{3}$$

at the feed point, $Z = Z_F$, and Eq. 2 becomes

$$X_{\mathrm{FEED}} = K_1\, e^{U_x Z_F/De} + K_2 \tag{4}$$

Using fixed concentration boundary conditions at the top of the elutriator and at the feed point leads to the solution for the concentration profile as follows:

$$X = \frac{X_{\mathrm{FEED}} - X_{\mathrm{TOP}}}{e^{U_x Z_F/De} - 1}\left[e^{U_x Z/De} - 1\right] + X_{\mathrm{TOP}} \tag{5}$$

To check whether this equation predicts concentration variation adequately, a comparison was made with experimentally determined concentration profiles determined from photographic studies. Figure 2 shows a comparison using 6-mm diameter uniform glass spheres, using a value for eddy diffusivity from the values of Levenspiel (1972) for single-phase dispersion coefficients. The comparison can be seen to be quite encouraging, and this will be referred to again later.

The conditions at the bottom of the elutriator are obtained by equating the convective flow just outside the bottom of the tube with the flow due to convection and backmixing just inside the apparatus (Mecklenburgh and Hartland, 1975). This leads to a prediction of a concentration gradient of zero below the feed zone for the heavy species. This has been observed experimentally, Figure 2, and results in the following equation for conditions just above the feed zone:

$$\left.\frac{dX}{dZ}\right|_{Z=Z_{\bar{F}}} = \frac{F_H}{De} \tag{6}$$

The rate of loss of heavy species particles from the top of the elutriator can be shown by using Eqs. 3 and 4, together with an overall mass balance and Eq. 6, to be:

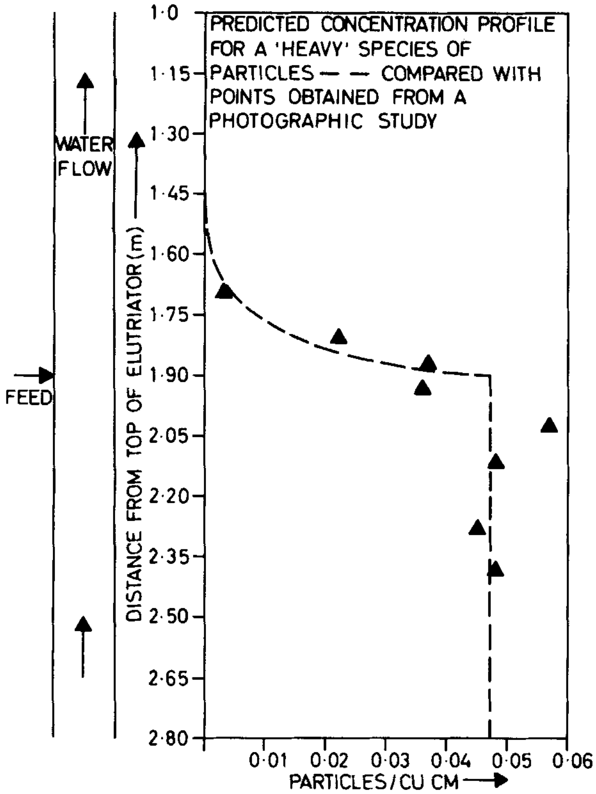

Figure 2. Predicted and measured concentration profiles.

$$T_H = V_w X_{TOP} = \frac{F_H}{\left(\frac{U_x}{V_w} - 1\right)_e \frac{U_x Z_F}{De_H}} \tag{7}$$

The rate of loss of heavy species from the bottom is given by:

$$B_H = U_x (X_F)_H = F_H \left[1 - \frac{1}{\left(1 - \frac{U_x}{V_w}\right)_e \frac{U_x Z_F}{De_H}} \right] \tag{8}$$

For the light species, it is assumed that the fluid velocity just below the bottom of the elutriator tube is zero, and the analogous equations are:

$$B_L = \frac{F_L \, e^{U_x Z_B / De_L}}{\left(1 - \frac{U_x}{U_t}\right)_e \frac{U_x Z_F}{De_L}} \tag{9}$$

$$T_L = U_x (X_F)_L = F_L \left[\frac{e^{U_x Z_B / De_L}}{\left(1 - \frac{U_x}{U_t}\right)_e \frac{U_x Z_f}{De_L}} - 1 \right] \tag{10}$$

Equations 7–10 allow the product rates to be predicted for given feed rates under the conditions assumed in the theoretical model. Measurements of concentration variations for both light and heavy species, similar to that illustrated in Figure 2, have confirmed the predictions from the model.

No account has been taken of any interactions between particles in this simple model. At high solids feed rates this could clearly become important, and so the present model is more likely to be useful in situations of moderate solids feed rates.

A separation factor (C) is defined to reflect the success of the separation. This is defined as

$$C = A_H + (1 - A_L) \tag{11}$$

This separation factor has a value of zero for perfect separation and a value of unity for no separation. Any value between zero and unity represents partial separation.

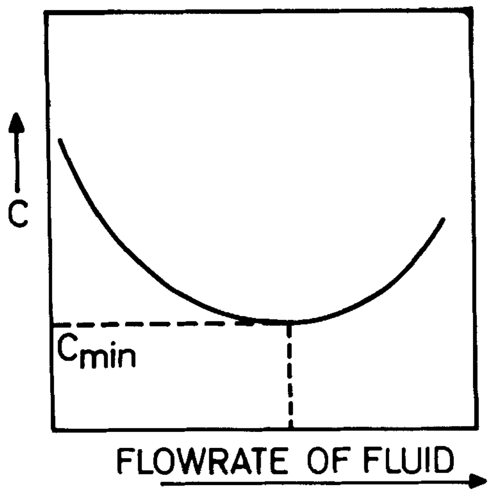

Figure 3. Criterion of separation as a function of fluid flowrate.

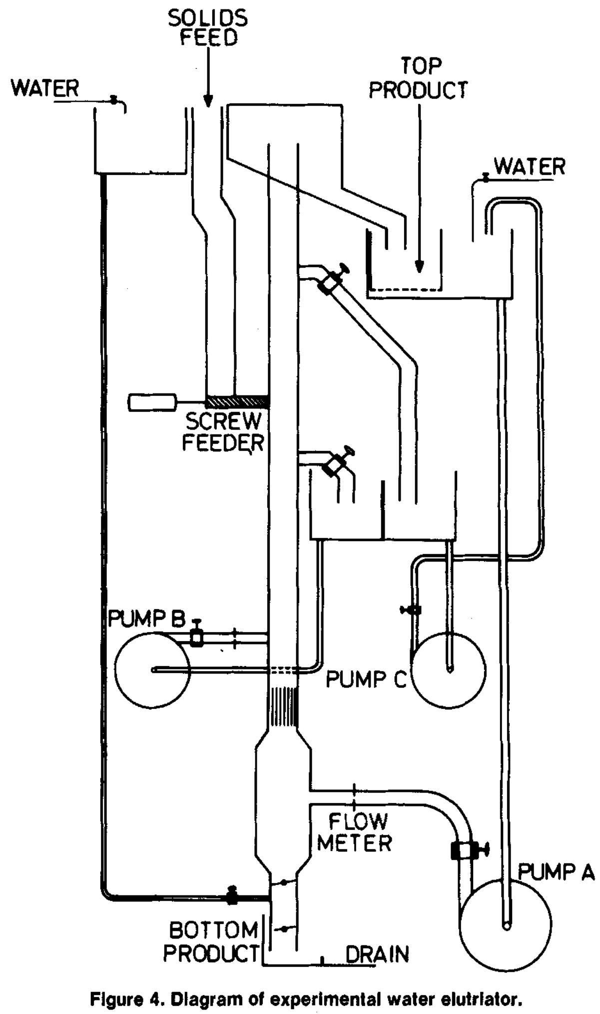

Figure 4. Diagram of experimental water elutriator.

The pattern of variation of C with fluid flow rate would be expected to be of the general form, Figure 3. At very low fluid flow rates all material sinks to the bottom. At very high flow rates all material is swept over the top. Between these extremes there exists a minimum value of C which indicates the optimum separation achievable in a particular separator. The value of C_{min} at this minimum point indicates the extent of separation. This will depend on the material characteristics and the design of the elutriator.

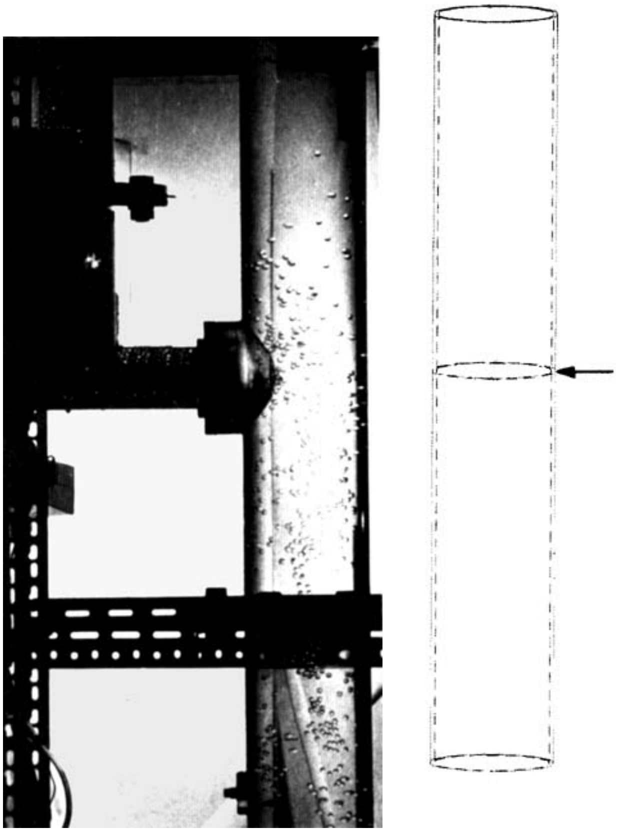

Figure 5. Comparison of actual distribution and predicted distribution of 6-mm glass spheres.

Experimental Equipment

A diagram of the equipment used in this investigation is shown in Figure 4. The main elutriator tube is 15 cm in diameter and 4.3 m high, constructed from clear, rigid polyvinylchloride. The water is pumped by the main pump (A) and passes through a control valve and a calibrated orifice plate meter before entering the bottom section of the elutriator. A flow-straightener removes swirling from the fluid, and the water passes up through the tube at average velocities up to 0.6 m/s. The water overflows the top of the tube and passes to a 0.5-m^3 storage tank before passing back to the inlet to the main pump. The solids mixture to be separated is fed into the hopper and down to a variable speed screw-feeder which injects the solids into the fluid. Any solids leaving the top of the tube are caught in a mesh-trap, and any leaving the bottom are similarly retained.

The equipment has the facility to inject an auxiliary fluid stream, from pump B, below the feed zone, and there are two fluid take-off points higher up the column. A pump C is provided to return fluid from a lower storage tank to the main storage tank. These auxiliary flow facilities have not been used in this study.

Results

To test the validity of the theoretical model for the operation of the water elutriator, standard mixtures of uniform particles were separated. One hundred kilograms of uniform glass spheres of diameters 3, 4 and 6 mm were obtained. The mixtures studies were 3 mm/4 mm and 4 mm/6 mm, in each case equal numbers of the smaller and larger particles being mixed. However, before any actual separations of mixtures were attempted, a photographic study of the operation of the elutriator using a single species of uniform particles was carried out to measure concentration distributions. Figure 5 shows the typical central region of the elutriator around the feed zone. By dividing the photographic print up into sections of known volume and counting particles it is possible to obtain concentration distributions. The particles used in this run were 6-mm glass spheres. The average terminal velocity in water had been determined by timing the fall of ten randomly selected spheres in still water. This average was found to be 0.5438 m/s, with a standard deviation of 0.008 m/s. This average value was close to the Stoke's Law prediction of 0.5442 m/s.

In Figure 5, the bulk average water velocity up the elutriator tube was known to be 0.41 m/s. This was measured using a carefully calibrated orifice plate meter and confirmed by a pitot tube measurement. Therefore, since the upward velocity of the water was significantly lower than the terminal falling velocity of the spheres, the predominant effect is a general downward flow of the glass spheres, leaving eventually from the bottom product offtake. The difference between the upward fluid velocity and the downward particle velocity enables the calculation of the number of particles which should be in any known volume below the feed zone. Residence time considerations lead to:

$$U_x = GH/N. \tag{12}$$

When this is applied to the conditions existing in Figure 5, a fluid velocity of 0.36 m/s is predicted, compared with that actually measured at 0.41 m/s. The effective fluid velocity in the elutriator is about 12% lower than the real, bulk average, fluid velocity.

Observation indicates that this is probably due to the tendency of the particles to cluster and migrate towards the wall, and hence towards regions of lower fluid velocity. This phenomenon of wall flow will be discussed later.

Now consider the region above the feed point. It can be seen that the particles have penetrated significantly upwards, in spite of the fact that they are really trying to move downwards. This is a consequence of turbulent mixing and is taken into account in the mixing theory. The extent of penetration upwards depends on the value of eddy diffusivity (De_H) which can be expected. In an earlier paper (Biddulph, 1979) based on work on a much smaller elutriator, it was established that eddy diffusivity values to be expressed in this situation were similar to those measured by Levenspiel (1972) for single-phase dispersion coefficients in pipe flow. If we use this method, we have the following equation for De_H (m^2/s):

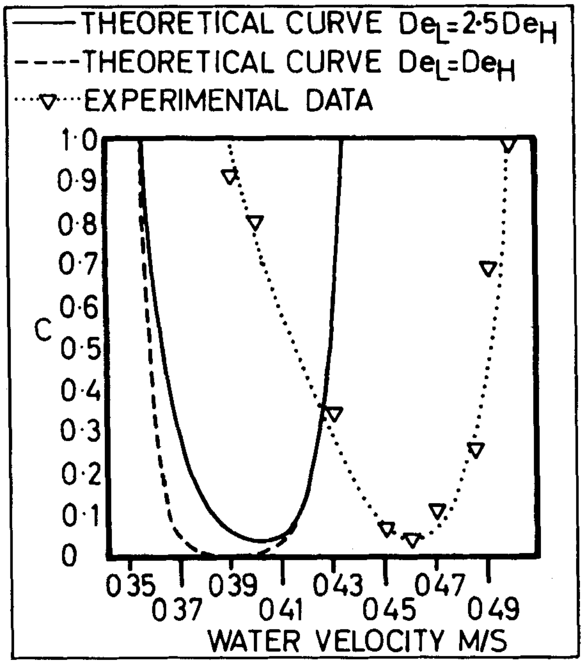

Figure 6. Predicted and measured values of *C* for a mixture of 3 mm/4 mm glass spheres.

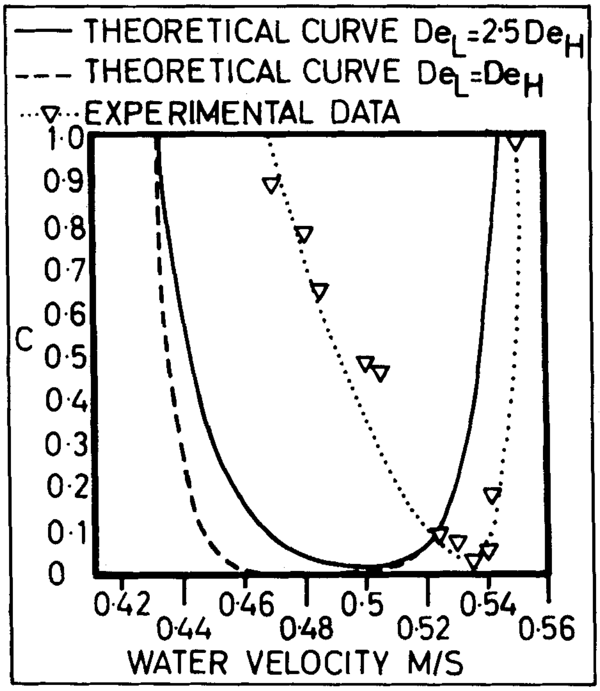

Figure 7. Predicted and measured values of *C* for a mixture of 4 mm/6 mm glass spheres.

$$De_H = V_w d_t \left[\frac{3.34 \times 10^7}{Re^2} + \frac{0.62 \times 10^3}{Re} + 0.22\right] \tag{13}$$

Building this into a computer simulation of the process results in the line shown in Figure 2. Although there is some scatter among the experimental points, it can be seen that the comparison is encouraging. Another way of comparing the theoretical model and the experimental observation is to program the computer to illustrate the expected appearance of the region near the feed zone. To do this, numbers of particles are predicted in various zones using Eq. 5, and these are randomly distributed throughout the fluid within the appropriate region of the column. When the particles have been positioned using a random number generator within the short cylindrical zones, the appearance when looking through the side of the column is predicted. This results in the appearance shown on the righthand side of Figure 5. It can be seen that the extent of penetration above the feed point is rather similar. It can also be seen that there is more clustering in the real case, compared with completely random distribution.

At higher fluid velocities it is possible for a heavy species, that are predominantly moving downwards, nevertheless, to penetrate upwards to such an extent that it appears in the top product.

A series of photographic investigations were carried out at various fluid velocities and for glass spheres of diameters 3, 4 and 6 mm. It was found that the Levenspiel value of eddy diffusivity always gave good predictions of the penetration of a heavy species upwards. It was also found that the "effective" fluid velocity was always 5–15% lower than the actual fluid velocity.

When the same series of runs were carried out but with the upwards fluid velocity being greater than the single species particle terminal velocity, it was found as expected that the predominant direction of movement was upwards. Again, the "effective" fluid velocity was always slightly lower than the real fluid velocity. However, it was also found that the penetration of the light species downwards due to mixing was much greater than would be predicted by using the Levenspiel values for De_L. This was again observed to be clearly due to clusters of the particles gathering near the wall and moving downwards some distance before being dispersed into the bulk of the fluid. Values of eddy diffusivity of 2 or 3 times the Levenspiel value were required to predict the extent of downwards penetration.

Separation of Mixtures

Two different, standard mixtures were separated over a range of fluid velocities. These consisted of equal numbers of 3- and 4-mm diameter glass spheres and equal numbers of 4- and 6-mm-diameter glass spheres. In each case a series of runs were carried out with the mixture being fed into the elutriator at a known rate for 30 minutes and the top and bottom products being collected. These products were then analyzed by sieving and weighing. The value of the separation factor (C) was calculated. The results for the 3/4-mm mixture are shown in Figure 6. It can be seen that the curve through the experimental points show the same trend as predicted by the theory, a minimum value of C occurring and representing optimum separation. Also shown on the same diagram are the theoretically predicted curves using: a) the Levenspiel value for De_H and 2.5 times the Levenspiel value for De_L; and b) the Levenspiel value for both species.

It can also be seen that the experimental curve is displaced to the right of the theoretical curve, this being due to the effect of wall flow of particles making the measured bulk average fluid velocity greater than the effective fluid velocity. It can also be seen that both the experimental and the theoretical curve (a) are skewed to the right, and this is also caused by the influence of wall flow. This is because the wall flow affects the bottom product particularly and creates asymmetry in the separation factor curve.

The analogous results for the 4/6-mm-diameter mixture of glass spheres is shown in Figure 7. It can be seen that the same effects are found. The theoretical curves were again produced in the same way. The displacement to the right is not quite as marked in this case, the higher fluid velocities apparently discouraging wall flow.

The results give encouragement that the behavior of a water elutriator may be simulated, at least for standard mixtures, using a relatively simple mathematical model. They also highlight the presence and influence of wall flow of the particles, and in particular the result that a vertical elutriator of this type is good at producing a pure top product but less successful at producing a bottom product of high purity. This indicates the desirability of placing the solids feed injection point nearer the top than the bottom of the vertical tube, and the model will enable optimization of the feed location.

The results of this study so far have clarified some effects, and there may be a number of ways to overcome the problems and hence improve the separating efficiency of water elutriators operating in this region of velocity.

Firstly, attempts could be made to eliminate wall flow by forcing descending particles away from the wall and into the bulk fluid. This may be possible by incorporating discontinuities in the wall. Alternatively, a local region of increased fluid velocity may be introduced below the feed point by using an auxiliary pump to inject extra fluid in addition to the main fluid flow, the extra fluid being taken off as a sidestream immediately below the solids feed point. These possibilities will be incorporated in the next phase of the study, followed by a study of the separation of real solids waste mixtures containing a range of particle sizes and shapes.

NOTATION

A = number of fraction of the feed of a particle species leaving the top product
B = rate of loss of a species from bottom (particles/m^2·s)
C = separation factor
C_{min} = optimum value of separation factor
d_t = tube diameter (m)
De = eddy diffusivity (m^2/s)
F = feed rate of a species (particles/m^2·s)
G = feed rate of particles (particles/s)
H = height of a test zone (m)
K_1,K_2 = constants
N = number of particles in a test zone
T = rate of loss of a species from the top (particles/m^2·s)
U_t = terminal falling velocity in stationary fluid (m/s)
U_x = net velocity of a species X (m/s)
V_W = mean water velocity (m/s)
X = concentration of species X (particles/m^3)
Z = distance from top of elutriator (m)
Z_F = distance of feed from top of elutriator (m)
Re = fluid tube Reynolds Number

Subscripts

H = heavy species
L = light species
F = feed zone
B = bottom

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Separation of Benzene and Toluene from Close Boiling Nonaromatics by Extractive Distillation

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Benzene and toluene are virtually impossible to separate from close boiling nonaromatic hydrocarbons by rectification. Benzene and toluene can be readily separated from similar boiling nonaromatics by using extractive distillation in which the extractive distillation agent is a proper mixture of organic compounds boiling higher than benzene or toluene. A typical extractive distillation agent for benzene is a mixture of phthalic anhydride, maleic anhydride, and adiponitrile; for toluene, phthalic anhydride, maleic anhydride, and glycerol triacetate.

SCOPE

Extractive distillation is the name applied to the method of altering the relative volatility of two or more compounds by distilling them in the presence of a quantity of a higher boiling liquid. Typically the extractive distillation agent boils at least 20°C higher than the compounds being separated, forms no azeotrope with them, and is miscible with them at distilling temperature. It was the objective of this research to find extractive distillation agents which will increase the apparent relative volatility of benzene or toluene to similar boiling nonaromatic hydrocarbons to a value high enough to permit separation in a rectification column having a moderate number of theoretical plates thus making rectification an economically